

Roger LeB. Hooke

# PRINCIPLES OF GLACIER MECHANICS

THIRD EDITION





# Principles of Glacier Mechanics

The third edition of this successful textbook will supply advanced undergraduate and graduate students with the tools they need to understand modern glaciological research. Practicing glacial geologists and glaciologists will also find the volume useful as a reference book. Since the second edition, three-quarters of the chapters have been updated, and two new chapters have been added. Included in this edition are noteworthy new contributions to our understanding of important concepts, with over 170 references to papers published since the second edition went to press. The book develops concepts from the bottom up: a working knowledge of calculus is assumed, but, beyond that, the important physical concepts are developed from elementary principles. Emphasis is placed on connections between modern research in glaciology and the origin of features of glacial landscapes. Student exercises are included.

**Roger LeB. Hooke** is Research Professor in the School of Earth and Climate Sciences and the Climate Change Institute at the University of Maine. He has been involved in glaciological research for over 30 years, focusing on processes relevant to the origin of glacial landforms. In addition to the first and second editions of *Principles of Glacier Mechanics*, he has published over 100 research papers in journals such as the *Geological Society of America Bulletin*, *Geology*, the *Journal of Glaciology*, *Quaternary Research*, and the *Journal of Geology*.

“*Principles of Glacier Mechanics* by Roger Le B. Hooke is a must-have for anyone seriously interested in glaciers and ice sheets. This 3rd edition provides a compact, accessible, rigorous perspective on the last few decades of evolution in our understanding of glacier mechanics, and connects the reader from basic, fundamental principles to the most recent research.”

– Eric Rignot, University of California–Irvine

“This is the first book I’d recommend to a student or colleague who wants to understand the fundamentals of how glaciers work. It’s a fantastic textbook for teaching glaciology to senior undergraduate and graduate students in the geosciences. Painstaking efforts are made to instill conceptual understanding of processes before developing mathematical understanding. The book is truly aimed at teaching, rather than simply informing, and it succeeds admirably. More so than any other text, it lucidly establishes connections between the mechanics of glaciers and the spectacular landforms they create. The third edition of the book is more comprehensive than the first two editions, with additional chapters on ice streams/shelves and ice cores – two of the most topical and important subjects in glaciology. These additional chapters add significantly to its great value as an authoritative reference book. The lean, crisp writing and emphasis on building understanding from the bottom up make this an unusually readable introduction to a subject with increasing societal relevance as the climate warms.”

– Neal Iverson, Iowa State University

“Today, glaciology is one of the cornerstones of the Earth sciences. The book *Principles of Glacier Mechanics* provides an excellent overview of the subject and can be recommended for both students and professionals wanting to gain insight into this rapidly growing field. The book strikes a nice balance between the quantitative and qualitative aspects of glacier mechanics. The reader is provided with an excellent summary of observations of glaciers and ice sheets from around the world, and all the key physical principles and governing equations of glacier mechanics are presented and explained in a very accessible fashion. In short, this is a well-written and concise text on glacier mechanics and an excellent book for teaching and learning the mechanics of glacier flow.”

– G. Hilmar Gudmundsson, Northumbria University

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University of Maine



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It is with a deep sense of gratitude that I dedicate this book to those who, at various times through the formative stages of my life, guided me into the most exciting and rewarding career I can imagine: the study of our Earth.

To my parents, who opened many doors for me;

to my older brother, Richard, who led me through a door leading to the wilderness;

to John Muir, who opened my eyes to the spirituality in wilderness;

to my wife, Ann, who introduced me to Geology;

to John P. Miller, who focused my attention on processes at the Earth's surface; and

to Robert P. Sharp, who taught me that basic physical principles could be used to understand these processes.





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## Preface to the third edition

Eleven years after publication of the second edition of *Principles*, our understanding of glaciers and ice sheets had advanced to the point that a third edition seemed warranted. It has taken 3 years to bring this to fruition. As before, I thank my Cambridge editor, Matt Lloyd, for his patience.

My objectives in writing this book are detailed in the prefaces to the first and second editions: namely to introduce upper division and graduate students to the fundamentals of glaciology, and in so doing to perhaps provide a reference book of use to practicing glaciologists and to glacial geologists seeking to understand the formation of diverse glacial landforms. These objectives have not changed. In keeping with that goal, many advanced topics are left to more specialized works.

New in this edition are chapters on ice streams and ice shelves and on ice-core studies. These are areas of glaciology that are particularly topical today, as we worry about the effects of climate warming on ice sheets, and try to understand the past climate system.

In addition to those acknowledged for their help and encouragement in previous prefaces, I'd like to express my appreciation for assistance provided by David Bahr, Carolyn Begeman, Allison Banwell, David Goldsby, Hilmar Gudmundsson, Brian Hanson, Neal Iverson, Doug MacAyeal, Keith Makinson, Paul Mayewski, Stephen Price, Eric Rignot, Sharon Sneed, Dominique Reynaud, Gerard Roe, and Sebastian Rosier in the course of preparing this edition.

June 7, 2019





## Preface to the second edition

When I wrote the preface to the first edition of this book 7 years ago, nothing was further from my mind than a second edition. The first edition was well received, however, and on numerous occasions colleagues have lamented the fact that it was no longer available. When Cambridge University Press agreed that a new edition was desirable, little did I realize what I had gotten into.

When I told Matt Lloyd (my editor at Cambridge) that my goal was to have the text ready by a certain time, he graciously gave me a target date that was nearly double that time. I told him that his time schedule was fine, but that I did not want to be held too strictly to it. As it happens, I had an unrealistic view of the volume of new material that needed to be sifted through, absorbed, and translated into language appropriate for the upper-division undergraduate and graduate-level students for whom this book is written. As with the first edition, my goal is not to provide an encyclopedia of research in glaciology, as other books do that well, but rather to give students the basic background they will need to understand the modern literature. At the same time, the book has proven to be a useful reference for professionals who don't keep all of the equations and conversion factors stored for instant recall. I myself use it for that purpose frequently.

I am indebted to many who have encouraged me in this undertaking, and especially to those who have generously given their time to review new sections or entire chapters, who have resurrected archived computer files to provide images or data files from which new figures were produced, or who have made new calculations especially for this volume. The following have assisted me in this effort: Richard Alley, Bob Bindschadler, Ginny Catania, Chris Clark, Lee Clayton, Paul Cutler, Gordon Hamilton, Brian Hanson, Bruce Hooke, Peter Hudleston, Kolumbian Hutter, Philippe Huybrechts, Neal Iverson, Peter Jansson, Susan Kaspari, Katie Leonard, Paul Mayewski, Shawn Marshall, Howard Mooers, Nadine Nereson, Felix Ng, Charlie Raymond, Vandy Spikes, Slawek Tulaczyk, and Joe Walder.

March 15, 2004



## Preface to the first edition

One might well ask why one should write a book about so specialized a subject as glacier mechanics when there are already other good books on this subject written by eminent glaciologists. This book is an outgrowth of a course that I teach to students who, in many cases, do not have any background in continuum mechanics. Consequently, it was necessary to start at a level considerably less advanced than that at which other similar books begin, and to develop the theoretical principles one step at a time. Thus, unlike other books on the subject and the general scientific literature, in which space is at a premium, the steps leading from one equation to another are, in most cases, easily seen. In addition, qualitative interpretations of the equations are often provided to clarify the physics behind the mathematics. Capable students with a solid background in basic physics and in differential and integral calculus, and with some modest exposure to differential equations, will have little difficulty understanding the concepts and derivations presented.

My goal in writing this book was not to produce a comprehensive treatise on glacier mechanics, but rather to develop the basic foundation upon which the modern literature on this subject rests. Thus, many topics are not covered, or are treated in less detail than some readers might wish. However, students who have a full appreciation for the concepts in this book will have the background they need to understand most of the current literature.

Beginning students in glaciology will find that this book will save them many long hours of searching through the background literature to clarify basic concepts. Glacial geologists and geomorphologists will also find much of value, including applications of glacier physics to the origin of some glacial landforms. Structural geologists and others with interest in stress and deformation will likewise discover that glaciers are, in fact, monomineralic rock masses that are deforming at the Earth's surface where they can be observed in detail. The book is, thus, appropriate for upper division and graduate level courses in glaciology, and as a supplementary text for courses in glacial geology and in structural geology.

In the preliminary pages, readers will find a compilation of physical constants relevant to ice, and a list of SI units and conversion factors. A series of problems keyed to individual chapters is also included.

The encouragement I have received in this undertaking from many present and former students, as well as from other glaciologists, has been a major stimulus in bringing it to completion. I trust the final product is worthy of their confidence. The book has benefited from the critical comments of R. W. Baker at the University of

Wisconsin, River Falls; C. R. Bentley at the University of Wisconsin, Madison; G. K. C. Clarke at the University of British Columbia; E. M. Grace and B. Hanson at the University of Delaware; N. R. Iverson at the University of Minnesota; T. Jóhannesson at the Icelandic Meteorological Office; M. Kuhn at the University of Innsbruck, Austria; M. F. Meier at the University of Colorado; J. F. Nye at the University of Bristol, England; C. F. Raymond at the University of Washington; R. L. Shreve at the University of California, Los Angeles; J. Weertman at Northwestern University, and especially I. Whillans at Ohio State University.

June 25, 1996

# Physical constants relevant to ice

Symbol	Parameter	Value
a	Coefficient of linear thermal expansion of: ordinary water at 0°C ice at -10°C	-22.3 × 10 <sup>-6</sup> K <sup>-1</sup> 51.6 × 10 <sup>6</sup> K <sup>-1</sup>
b	Burgers vector	4.5 × 10 <sup>-10</sup> m
C	Heat capacity of pure ice at 0°C C varies with temperature, approximately thus: C = 152.5 + 7.122θ, where θ is in Kelvins (Cuffey and Paterson, 2010, p. 400). For more detailed data see Yen (1981)	2096 J kg <sup>-1</sup> K <sup>-1</sup>
C <sub>w</sub>	Heat capacity of air-free water at constant pressure and 0°C	4184 J kg <sup>-1</sup> K <sup>-1</sup>
C	Depression of the melting point Pure ice and air-free water Pure ice and air-saturated water (Harrison, 1972)	0.074 K MPa <sup>-1</sup> 0.098 K MPa <sup>-1</sup>
E	Young's modulus (Gold, 1958) [The ratio of axial stress to elastic axial strain in a test in uniaxial tension. E = 2μ(1 + ν)]	8.3 × 10 <sup>3</sup> MPa*
g	Acceleration of gravity	9.81 m s <sup>-2</sup>
K	Thermal conductivity at -1°C K varies with temperature, thus: K = 7.10 × 10 <sup>7</sup> - 0.0195 × 10 <sup>7</sup> θ + 0.000363 × 10 <sup>7</sup> θ <sup>2</sup> where θ is the temperature in °C (a negative number) (Ratcliffe, 1962)	7.1 × 10 <sup>7</sup> J m <sup>-1</sup> a <sup>-1</sup> K <sup>-1</sup>
K	Bulk modulus (at -5°C) (Gold, 1958) (Ratio of applied pressure to fractional change in volume)	8.7 × 10 <sup>3</sup> MPa*
K <sub>IC</sub>	Fracture toughness (Rist <i>et al.</i> , 1999)	0.05–0.15 MPa m <sup>1/2</sup>
L	Heat of Fusion	3.34 × 10 <sup>5</sup> J kg <sup>-1</sup>
Q	Activation energy for creep below -10°C Q appears to vary with stress (Goldsby and Kohlstedt, 1997), with 60 kJ mol <sup>-1</sup> being a good average value at stresses commonly found in glaciers. Above -10°C, Q is presumably the same but the ε̇ vs 1/θ curve steepens due to the presence of a liquid phase on grain boundaries	60 ± 10 kJ mol <sup>-1</sup>
V	Activation volume (Kirby <i>et al.</i> , 1987)	-13 × 10 <sup>-6</sup> m <sup>3</sup> mol <sup>-1</sup>
R	Gas constant	8.314 J mol <sup>-1</sup> K <sup>-1</sup>
S <sub>cr</sub>	Crushing strength of natural snow ice.	1.8 MPa at 0°C

(cont.)		
Symbol	Parameter	Value
	The strength increases substantially with decreasing temperature. Hobbs (1974, p. 331) gives a graph from Butkovitch (1954) that can be approximated by: $S_{cr} = 1.8 - 0.266\theta - 0.0202\theta^2 - 7.72 \times 10^{-4}\theta^3 - 1.39 \times 10^{-5}\theta^4 - 9.37 \times 10^{-8}\theta^5$ where $\theta$ is the temperature in $^{\circ}\text{C}$ (a negative number) There is considerable variability depending on the type of ice tested and its orientation.	
$\beta$	Dihedral angle ( $\text{Cos } \beta = \gamma_{gb}/2\gamma_{SL}$ ) (Nye and Mae, 1972)	$2\beta = 32 \pm 3^{\circ}$
$\gamma_{SL}$	Specific surface energy of liquid–solid interface (Ketcham and Hobbs, 1969)	$0.034 \text{ J m}^{-2}$
$\gamma_{gb}$	Specific surface energy of grain boundary	$0.065 \text{ J m}^{-2}$
$\theta_m$	Melting point at atmospheric pressure	$0.0^{\circ}\text{C}$ $273.15 \text{ K}$
$\theta_{TP}$	Triple point temperature	$+0.0098^{\circ}\text{C}$
$P_{TP}$	Triple point pressure	$600 \text{ Pa}$
$\kappa$	Thermal diffusivity at $-1^{\circ}\text{C}$ [Below $-0.5^{\circ}\text{C}$ , $\kappa$ varies with temperature due to the variation in $K$ (see above). Above $-0.5^{\circ}\text{C}$ , $\kappa$ decreases due to the increase in effective $C$ (see above). Paterson (1971) estimates that, at $-0.1^{\circ}\text{C}$ , $\kappa$ is half its value for pure ice, and at $-0.01^{\circ}\text{C}$ it is 1% of the value for pure ice. These estimates assume a salinity of $10^{-6}$ ]	$37.2 \text{ m}^2 \text{ a}^{-1}$
$\mu$	Shear modulus (at $-5^{\circ}\text{C}$ ) (Gold, 1958) [The ratio of shear stress to elastic shear strain in a test in simple shear]	$3.8 \times 10^3 \text{ MPa}^*$
$\nu$	Poisson's ratio for polycrystalline ice (Gold, 1958) [The ratio of the transverse strain (contraction) to the axial strain (extension) of a bar in a uniaxial tensile test]	$0.31^*$
$\rho_i$	Density of bubble-free ice	$916 \text{ kg m}^{-3}$
$\rho_w$	Density of water at $0^{\circ}\text{C}$	$999.84 \text{ kg m}^{-3}$
$\zeta$	Depression of melting point due to solutes	$1.86^{\circ}\text{C kg mol}^{-1}$

\* Values given for  $E$ ,  $K$ ,  $\mu$ , and  $\nu$  are based on the work of Gold (1958), as reported by Hobbs (1974, pp. 255–258). Hobbs also reports other values based on the work of other (earlier) investigators.

## Derived SI units and conversion factors

$1 \text{ N} = 1 \text{ kg m s}^{-2}$	Force (mass · acceleration)
$1 \text{ Pa} = 1 \text{ N m}^{-2} = 1 \text{ kg m}^{-1} \text{ s}^{-2}$	Stress
$1 \text{ J} = 1 \text{ N m} = 1 \text{ kg m}^2 \text{ s}^{-2}$	Work or energy
$1 \text{ W} = 1 \text{ J s}^{-1} = \text{N m s}^{-1}$	Power
$1 \text{ bar} = 0.1 \text{ MN m}^{-2} = 0.1 \text{ MPa} = 0.9868 \text{ atm}$	Stress
$1 \text{ cal} = 4.18 \text{ J}$	
$1 \text{ a} = 3.15569 \times 10^7 \text{ s}$	
$0^\circ\text{C} = 273.15 \text{ K}$	





# 1

## Why study glaciers?

Before delving into the mathematical intricacies with which much of this book is concerned, one might well ask why we are pursuing this topic – glacier mechanics? For many people who would like to understand how glaciers move, how they sculpt the landscape, how they respond to climatic change, mathematics does not come easily. I assure you that all of us have to think carefully about the meaning of the expressions that seem so simple to write down but so difficult to understand. Only then do they become part of our vocabulary. Only then can we make use of the added precision which mathematical analysis, properly formulated, is able to bring. Is it worth the effort? That depends upon your objectives; on why you chose to study glaciers.

There are many reasons, of course. Some are personal, some academic, and some socially significant. To me, the personal reasons are among the most important: glaciers occur in spectacular areas, often remote, that have not been scarred by human activities. Through glaciology, I have had the opportunity to live in these areas; to drift silently in a kayak on an ice-dammed lake in front of our camp as sunset gradually merged with sunrise on an August evening; to marvel at the northern lights while out on a short ski tour before bedtime on a December night; and to reflect on the meaning of life and of our place in nature. Maybe some of you will share these needs, and will choose to study glaciers for this reason. I have found that many glaciologists do share them, and this leads to a comradeship that is rewarding in itself.

Academic reasons for studying glaciers are perhaps difficult to separate from socially significant ones. However, in three academic disciplines, the application of glaciology to immediate social problems is at least one step removed from the initial research. The first of these is glacial geology. Glaciers once covered 30% of the land area of Earth, and left deposits of diverse shape and composition. How were these deposits formed, and what can they tell us about the glaciers that made them? The second discipline is structural geology; glacier ice is a metamorphic rock that can be observed in the process of deformation at temperatures close to the melting point. From study of this deformation, both in the laboratory and in the field, much has been learned about the origin of metamorphic structures in other crystalline rocks that were deformed deep within the Earth. The final discipline is paleoclimatology.

Glaciers record climatic fluctuations in two ways: the deposits left during successive advances and retreats provide a coarse record of climatic change which, with careful study, a little luck, and a good deal of skill, can be placed in correct chronological order and dated. A more detailed record is contained in ice cores from polar glaciers such as the Antarctic and Greenland ice sheets. Isotopic and other chemical variations in these cores reflect past atmospheric circulation patterns, changes in temperature, and changes in the composition of the atmosphere. Changes during the past several centuries to several millennia can be quite precisely dated using core stratigraphy. Those further back in time are dated less precisely using flow models and proxy measures of other well-dated phenomena such as Earth's orbital variations.

Relatively recent changes in climate and in concentrations of certain anthropogenic substances in the atmosphere are attracting increasing attention as humans struggle with problems of maintaining a healthy living environment in the face of overpopulation and the resulting demands on natural resources. Studies of ice cores and other dated ice samples provide a baseline from which to measure these anthropogenic changes. For example, levels of lead in the Greenland ice sheet increased about 4-fold when Greeks and Romans began extracting silver from lead sulfides in ~500 BCE (Hong *et al.*, 1994). Then, after dropping slightly in the first millennium AD, they increased to more than 80 times natural levels during the industrial revolution and to more than 200 times natural levels when lead additives became common in gasoline in ~1940 (Murozumi *et al.*, 1969). These studies are largely responsible for the fact that lead is no longer used in gasoline. Similarly, measurements of CO<sub>2</sub> and CH<sub>4</sub> in ice cores have documented levels of these greenhouse gases in pre-industrial times.

Other applications of glaciology are not hard to find. Some people in northern and mountainous lands live so close to glaciers that their lives would be severely altered by significant ice advances. Tales from the seventeenth and eighteenth centuries, a period of ice advance as the world entered the Little Ice Age, tell of glaciers gobbling up farms and farm buildings. Buildings were crushed into small pieces and mixed with "soil, grit, and great rocks" (Grove, 1988, p. 72). The Mer de Glace in France presented a particular problem, and several times during the seventeenth century exorcists were sent out to deal with the "spirits" responsible for its advance. They appeared to have been successful, as the glaciers were then near their Little Ice Age maxima and beginning to retreat. Increasing amounts of industrial black carbon, an aerosol, were falling on glacier surfaces then, absorbing solar radiation and increasing melt rates (Painter *et al.*, 2013)

Retreat may also present a problem. In many places, melting glacier ice provides a steady source of water for irrigation and other uses during the summer months. Glacier retreat reduces this flow and may divert it to a different valley. In the

western Himalaya, such a diversion forced the inhabitants of Kumik to move their village to a new location and dig a 7 km canal to provide water (Mingle, 2015).

Retreat of the Greenland and Antarctic Ice Sheets, together with that of numerous mountain glaciers world-wide, is also raising sea level. This retreat is expected to continue and to accelerate (Straneo and Heimbach, 2013) as global warming, exacerbated by black carbon from forest fires and burning of fossil fuels, increases melting. In the Admundsen Sea sector of West Antarctica, retreat of Thwaites and Pine Island glaciers could trigger collapse of the West Antarctic Ice Sheet, raising sea level ~3 m during the coming centuries to millennia (Park *et al.*, 2013; Feldmann and Levermann, 2015). In short, sea-level rise will increasingly impact our coastal infrastructure. Some political jurisdictions have had the foresight to begin planning for this eventuality.

Other people live in proximity to rivers draining lakes dammed by glaciers. Some of the biggest floods known from the geologic record resulted from the failure of such ice dams, and smaller floods of the same origin have devastated communities in the Alps and Himalayas. Somewhat further from human living environments, glaciologists may study the possibility of extracting economically valuable deposits from beneath glaciers, or how to curb the discharge of icebergs into shipping lanes.

Glacier ice itself is an economically valuable deposit; glaciers contain 60% of the world's fresh water, and peoples in arid lands have seriously studied the possibility of towing icebergs from Antarctica to serve as a source of water. People in mountainous countries use glacier meltwater not only for drinking, but also as a source of hydroelectric power. By tunneling through the rock under a glacier and thence up to the ice-rock interface, they trap water at a higher elevation than would be possible otherwise, and thus increase the energy yield. Glaciologists provide advice on the activity of the glaciers and where to find streams beneath them.

Lastly, we should mention a proposal to dispose of radioactive waste by letting it melt its way to the base of the Antarctic Ice Sheet. How long would such waste remain isolated from the biologic environment? How would the heat released affect the flow of the ice sheet? Might it cause a surge. In the end, this project was abandoned, not on glaciological grounds but, rather, because there seemed to be no risk-free way to transport the waste to Antarctica.

A good quantitative understanding of the physics of glaciers is essential for rigorous treatment of many of these academic problems, as well as for accurate analysis of various engineering and environmental problems involving glaciers and of concern to humans. The fundamental principles upon which this understanding is based are those of physics and, to a lesser extent, chemistry. Application of these principles to glacier dynamics is initially straightforward, but, as with many problems, the better we seek to understand the behavior of glaciers, the more involved, and often the more interesting the applications become.

## Why study glaciers?

So we have answered our first question; we study glaciers for the same reasons that we study many other features of the natural landscape, but also for a special reason which I will try to impart to you, wordlessly, if you will stand with me looking over a glacier covered with a thick blanket of fresh powder snow to distant peaks, bathed in alpine glow, breathless from a quick climb up a steep slope after a day of work, but with skis ready for the telemark run back to camp. “Mäktig,” my companion said – powerful.

# 2

## Some basic concepts

In this chapter, I'll introduce some basic concepts that will be used frequently throughout this book. First, we'll review some commonly used classifications of glaciers by shape and thermal characteristics. Then we consider the mathematical formulation of the concept of conservation of mass and, associated with it, the condition of incompressibility. This will appear again in Chapters 6 and 9. Finally, we discuss stress and strain rate, and lay the foundation for understanding the most commonly used flow laws for ice. Although a complete consideration of these latter concepts is deferred to Chapter 9, a modest understanding of them is essential for a fuller appreciation of some concepts presented in earlier chapters.

### A note on units and coordinate axes

SI (Système International) units are used in this book. The basic units of length, mass, and time are the meter (m), kilogram (kg), and second (s) (MKS). Temperatures are measured in Kelvins (K) or in the derived unit, degrees Celsius ( $^{\circ}\text{C}$ ). Some other derived units and useful conversion factors are given on p. *xxi*.

In most developments herein, I'll use a rectangular coordinate system with the  $x$ -axis horizontal or subhorizontal and in the direction of flow, the  $y$ -axis horizontal and transverse, and the  $z$ -axis normal to the other two and thus vertical or slightly inclined to the vertical. Some derivations are easier to approach with the  $z$ -axis directed upward, while in others it is simpler to have the  $z$ -axis directed downward.

### Glacier size, shape, and temperature

As humans, one way in which we try to organize knowledge and enhance communication is by classifying objects into neat compartments, each with its own label. The natural world persistently upsets these schemes by presenting us with particular items that fit neither in one such pigeonhole nor the next, but rather have

characteristics of both; continua are the rule rather than the exception. This is as true of glaciers as it is of other natural systems.

One way of classifying glaciers is by shape. Herein, we will be concerned with only two basic shapes. Glaciers that are long and comparatively narrow, and that flow in basically one direction, down a valley, are called *valley glaciers*. When a valley glacier reaches the coast and interacts with the sea, it is called a *tidewater glacier*. (I suppose this name is appropriate even in circumstances in which the tides are negligible.) Valley glaciers that are very short, occupying perhaps only a small basin in the mountains, are called *cirque glaciers*. In contrast to these forms are glaciers that spread out in all directions from a central dome. These are called either *ice caps*, or, if they are large enough, *ice sheets*.

There is, of course, a continuum between valley glaciers and ice caps or ice sheets. For example, one commonly finds valley glaciers flowing outward from ice caps or ice sheets; this kind of valley glacier is usually referred to as an *outlet glacier*. However, the end members, valley glaciers and ice sheets, typically differ in other significant ways (see, for example, Figure 3.1). Thus, a classification focusing on these two end members is useful.

Glaciers are also classified by their thermal characteristics, although once again a continuum exists between end members. We normally think of water as freezing at 0°C, but may overlook the fact that, once all the water in a space is frozen, the temperature of the resulting ice can be lowered below 0°C as long as heat can be removed from it. Thus, the temperature of ice in glaciers in especially cold climates can be well below 0°C. We call such glaciers *polar glaciers*. More specifically, polar glaciers are glaciers in which the temperature is below the melting temperature of ice everywhere above the bed. Parts of the bed may be at the melting point, and parts below it. The presence of meltwater at the bed has dramatic consequences, both for glacier kinematics and for landform development. In Chapter 6, we will investigate the temperature distribution in such glaciers in some detail.

Glaciers that are not polar are either *polythermal* or *temperate*. Polythermal glaciers contain large volumes of ice that are cold, but also large volumes that are at the melting temperature. Most commonly, the cold ice is present as a surface layer, tens of meters in thickness, on the lower part of the glacier (the ablation area).

In simplest terms, a temperate glacier is one that is at the melting temperature throughout. However, the melting temperature,  $\theta_m$ , is not easily defined. As the temperature of an ice mass is increased toward the melting point, veins of water form along lines where three ice crystals meet (Figure 8.1). At the wall of such a vein:

$$\theta_m = \theta_{TP} - \mathbf{C}P - \frac{\theta_m \kappa \gamma_{SL}}{L \rho_i r_p} - \zeta \frac{s}{W} \quad (2.1)$$